1^{st} Spanish Fluid Mechanics Conference
Cádiz, June 19-22, 2022

Theory of turbulence augmentation across hypersonic shock waves in air

A. Cuadra,* C. Huete, and M. Vera
Grupo de Mecánica de Fluidos, Universidad Carlos III, Leganés, Madrid 28911, Spain

The interaction between a weakly turbulent free stream of air and a hypersonic shock wave is investigated theoretically by using linear interaction analysis (LIA). The perturbation-free jump conditions across the shock are computed using Combustion Toolbox, an in-house thermochemical code capable of capturing high-temperature phenomena such as dissociation, ionization, and recombination in multi-species mixtures, which are found to be dominant effects in hypersonic shocks in air. The formulation is developed in the limit in which the thickness of the thermochemical nonequilibrium region is assumed to be much smaller than the characteristic size of the shock wrinkles caused by turbulence. Similarly to our previous work [Physics of Fluids, 33(8), 086111 (2021)] which only accounted for vibrational and dissociation effects of single-species diatomic gases, the LIA results presented here for air indicate that the enstrophy, anisotropy, intensity, and turbulent kinetic energy (TKE) of the fluctuations are much more amplified through the shock than in the thermochemical frozen case. Moreover, the turbulent Reynolds number is also amplified across the shock at hypersonic Mach numbers in the presence of dissociation and vibrational excitation, as opposed to the attenuation observed in the thermochemical frozen case. However, multi-species effects reshape the curve of TKE to that where local peaks associate with the local maxima in the net variation of the concentrations of the major diatomic species in the mixture.

Hypersonic technologies have been investigated for more than six decades, but their attention has grown in the last decade due to the significant raise of geopolitical tensions among major powers [1]. In fact, the number of publications on hypersonics has been steadily increasing since the early 80s due to its renewed relevance in aeronautical, astronautical, and military applications [2].

When shock waves appear in hypersonic conditions, the intense compression of the gas leads to high temperatures that can activate complex thermochemical phenomena, such us vibrational excitation, dissociation, electronic excitation, and ionization. Turbulence can also play a key role at high Mach numbers, specially in low-altitude hypersonic flight because of the correspondingly larger Reynolds numbers of the airflow around the fuselage and/or at the air intake of supersonic combustion ramjets [3]. The interaction of hypersonic shocks with turbulence in air is addressed in this work using LIA.

Assuming that turbulence is comprised of small linear vorticity fluctuations and that downstream perturbations can be separated using Kovaznay's decomposition into vortical, entropic, and acoustic modes, normal-mode analysis is used to compute the amplitude of the post-shock perturbation modes as a function of the shock properties [4]. The analysis demands the linearization of the Rankine–Hugoniot (RH) curve, previously computed with Combustion Toolbox [5]. This thermochemical code includes routines to accurately solve processes involving strong changes in the dynamic pressure, such as steady state detonations and shock waves (see Fig. 1). Unlike our previous work [6] that addressed single-component symmetric diatomic gases, the RH curve of air cannot be expressed analytically in terms of fundamental parameters, such as the rotational, vibrational or dissociation characteristic temperatures. Nevertheless, the current approach benefits from the inclusion of more complex effects, such as the recombination into multi-species gases and ionization, thereby increasing the range of the application of the theory to Mach numbers beyond 10.

In addition to the assumptions in the standard LIA, the incorporation of thermochemical effects requires that the characteristic size of the shock wrinkles be much larger than the thickness of the thermochemical nonequilibrium region behind the shock. The accuracy of this approximation in practical hypersonic systems is expected to improve as the flight Mach number increases and the altitude decreases.

Consider first the problem of an undisturbed, normal shock wave in a cold, inviscid, irrotational, air stream. The pre-shock density, pressure, flow velocity, and enthalpy in the reference frame of the shock are denoted, respectively, as ρ_1, p_1, u_1, h_1. The corresponding flow variables in the post-shock gas are denoted as ρ_2, p_2, u_2, and h_2. The corresponding RH relations are

$$\frac{p_2}{p_1} = 1 - \frac{\rho_1 u_1^2}{p_1}\left(\frac{\rho_1}{\rho_2} - 1\right), \tag{1a}$$

$$h_2 = h_1 + \frac{u_2^2}{2}\left[1 - \left(\frac{\rho_1}{\rho_2}\right)^2\right]. \tag{1b}$$

These equations are supplemented with the ideal-gas equations of state (EoS) $p = \rho R_g T$, where R_g is the gas constant of the gas mixture, and the corresponding function of state for enthalpy, which depends on temperature and the mixture properties. Note that thermal enthalpy reduces to $\gamma/(\gamma-1)p/\rho$ for a calorically perfect gas, an assumption that cannot be hold in hypersonic conditions. In our case, h is modelled with use made of the NASA-9 coefficient polynomials database, which ranges up to 20000 K. Combustion Toolbox implements this database and is used to solve (1) together with the chosen EoS to give the jump conditions across the shock for given free stream conditions and shock intensity. For example, Fig. 1 shows the RH curve of air in standard conditions (T_1 = 300 K, p_1 = 1 atm). Because of endothermic effects, it is clearly seen how recombination and dissociation augment the compression ratio with respect to that of a thermochemical frozen gas. Results have been

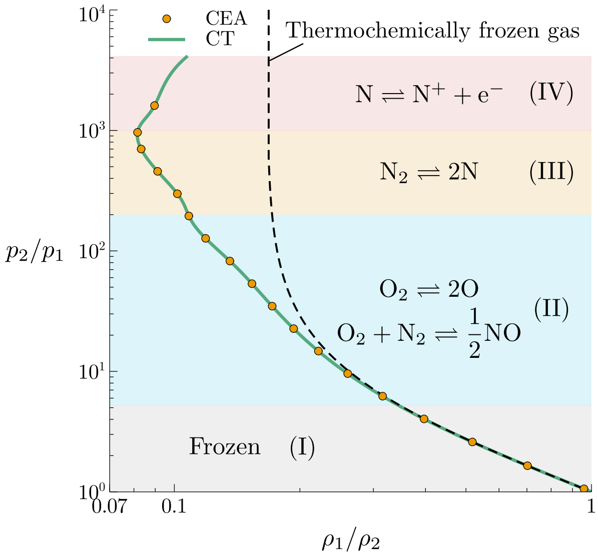

FIG. 1. Log-log RH curve for air at pre-shock temperature $T_1 = 300$ K, pressure $p_1 = 1$ atm, and volume composition $\{N_2, O_2, Ar, CO_2\} = \{78.08, 20.95, 0.9365, 0.0319\}$ including ionization; green line: Combustion Toolbox (CT) [5]; circles: numerical results obtained with NASA's Chemical Equilibrium with Applications (CEA) code [7]; Roman numerals: regions with the dominant reactions labeled.

compared with NASA's CEA code [7], showing excellent agreement.

The weak isotropic turbulence in the pre-shock gas can be represented by a linear superposition of incident vorticity waves whose amplitudes ε vary with the wavenumber in accord with an isotropic energy spectrum $E(k) = \varepsilon^2(k)$. The root mean square of the velocity and vorticity fluctuations in the pre-shock gas can be calculated by invoking the isotropy assumption, which states that the probability that the incident wave has a given orientation is proportional to the solid angle.

The TKE amplification factor across the shock wave is a magnitude of utmost interest in the interaction of shock waves with turbulence. It is defined as

$$K = \frac{\langle u'^2_2 \rangle + \langle v'^2_2 \rangle + \langle w'^2_2 \rangle}{\langle u'^2_1 \rangle + \langle v'^2_1 \rangle + \langle w'^2_1 \rangle} \tag{2}$$

where $\langle u'^2 \rangle$ denotes the mean value of the perturbation kinetic energy associated with velocity component u. By performing the theoretical analysis described in [6], with the details omitted here for brevity, the value of K can be expressed as an integral formula—corresponding to a isotropically weighted sum of contributions of the vorticity perturbation impacts on the planar shock—that ultimately depends on the post-shock properties (mass compression ratio, post-shock Mach number, and RH curve slope) computed with the aid of Combustion Toolbox. In our case, we have a multi-species mixture of gases composed mainly of O_2 and N_2, which have different characteristic dissociation temperatures.

The resulting curve for K (green line) is shown in Fig. 2. It is readily seen that it exhibits two peaks in regions (II) and (III). The non-monotonicity of K is dictated by the behavior of the vorticity generation across the shock, since acoustic turbulent kinetic energy is negligible in hypersonic conditions. In particular, two main effects are found to govern the post-shock perturbation flow: the mass compression ratio, whose amplification via endothermicity increases the flow deflection and the generation of transverse kinetic energy; and the RH curve slope, which is sensitive to the different inner processes undergoing within the non-equilibrium region. The latter is found to depend on the net rate of change of the molar fractions with the pre-shock Mach number $|dX_i/d\mathcal{M}_1|$ for the most important reactions in the mixture (inset).

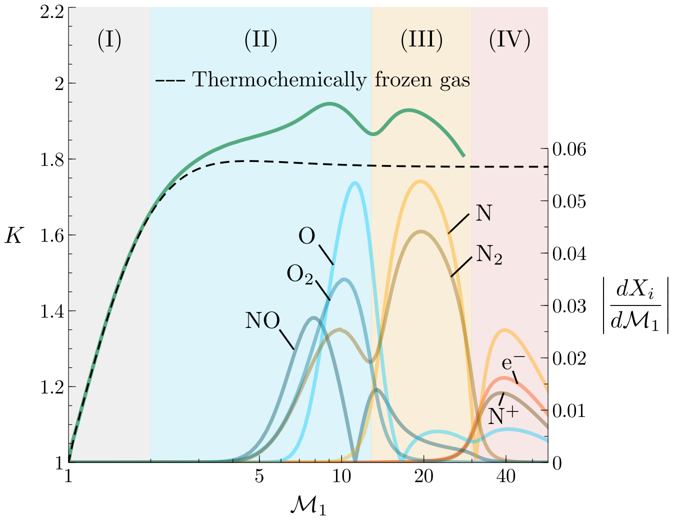

FIG. 2. TKE amplification factor K as a function of the pre-shock Mach number $\mathcal{M}_1$ (green line). Dashed line corresponds with the thermochemical frozen gas approximation. The inset represents the net rate change of the molar fractions with the pre-shock Mach number $|dX_i/d\mathcal{M}_1|$ for the most relevant reactions in the mixture.

---

* acuadra@ing.uc3m.es

[1] J. Mattis, Department of Defense Washington United States (2018).
[2] V. Theofilis, S. Pirozzoli, and P. Martin, Theoretical and Computational Fluid Dynamics, 1-8 (2022).
[3] J. Urzay and M. Di Renzo, Annual Research Briefs, Center for Turbulence Research, Stanford University, 7 (2020).
[4] H. S. Ribner, NACA Report 1233 (1954).
[5] A. Cuadra, C. Huete, and M. Vera, Zenodo, v0.7.3 (2022). DOI: 10.5281/zenodo.6168976.
[6] C. Huete, A. Cuadra, M. Vera, and J. Urzay, Physics of Fluids, 33(8), 086111 (2021).
[7] S. Gordon and B. J. McBride, NASA Reference Publication 1311 (1994).